

# “Back of the Envelope” Calculation of Climate Sensitivity

Andy Long

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## 1 Climate Sensitivity Model

Consider the problem of modelling Temperature  $T$  as a function of atmospheric carbon  $C$ , given climate sensitivity  $\alpha$  (increase in temperature given a doubling of carbon), and an historic temperature of  $T_o$  and historic CO<sub>2</sub> levels of  $C_o$ .

“Climate sensitivity” assumes a constant increase in temperature given a doubling of carbon dioxide, which leads to a logarithmic relationship:

$$T(2C) = T(C) + \alpha \tag{1}$$

and

$$T(C_o) = T_o \tag{2}$$

A logarithm will have the desired property of Equation (1):

$$T(C) = a \ln(C) + b$$

with

$$T(2C) = a \ln(2C) + b = a \ln(C) + a \ln(2) + b = T(C) + a \ln(2)$$

Hence, from Equation (1)

$$\alpha = a \ln(2)$$

or

$$a = \frac{\alpha}{\ln(2)}$$

Consequently

$$T(C) = \frac{\alpha \ln(C)}{\ln(2)} + b$$

and from Equation (2) we obtain

$$T(C_o) = \frac{\alpha \ln(C_o)}{\ln(2)} + b = T_o,$$

and can solve for  $b$ :

$$b = T_o - \frac{\alpha \ln(C_o)}{\ln(2)}$$

Hence

$$T(C) = T_o + \frac{\alpha}{\ln(2)} \ln\left(\frac{C}{C_o}\right) \quad (3)$$

## 2 Using the Model with Data to Estimate Sensitivity

So the historic level of CO<sub>2</sub> is reputedly  $C_o = 280$  ppm, whereas the historic temperature was about .8°C cooler.<sup>1</sup> The website suggests that CO<sub>2</sub> has risen by 42% since its historical values, so the computed “current” CO<sub>2</sub> would be 397.6 ppm.

Thus we can figure that

$$0.8 = T(C) - T_o = \frac{\alpha}{\ln(2)} \ln\left(\frac{397.6}{280}\right)$$

or

$$\alpha \approx 0.8 * \frac{\ln(2)}{\ln(1.42)} \approx 1.58 \quad (4)$$

So our estimate (based on the data on the website cited) for climate sensitivity is 1.58°C for a doubling of CO<sub>2</sub>. This ignores other gases that have also been rising, of course, and other effects (such as changes in albedo, ocean absorption, deforestation, etc.).

Hence  $TC$  which the IPCC (and the rest of the world, for that matter) says we should keep below 2°C, is simply a function of current carbon level  $C$

$$TC = T(C) - T_o = 2.28 \ln\left(\frac{C}{280}\right) \quad (5)$$

and we can solve this to find various important levels of CO<sub>2</sub>. For example, at what level will we hit 2°C of warming? The general formula is

$$C = 280 * e^{TC/2.28}$$

and so for These numbers would generally be considered gross overestimates of what carbon

Table 1: Temperatures obtained from various levels of CO<sub>2</sub> (ppm).

Temperature Increase	CO <sub>2</sub> (ppm)
2	673
3	1043
4	1618

pollution we can get away with. They come from deducing such a low level of climate sensitivity.

There are many other things going on. But this is a simple “back of the envelope” calculation.

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<sup>1</sup><http://www.skepticalscience.com/co2-temperature-correlation.htm>